TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 4 2017

Section 1 Calculator Free

Trigonometry

STUDENT'S NAME SOLUTIONS

DATE: Thursday 29 June **TIME:** 55 minutes **MARKS:** 55

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, page of A4 notes

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

Determine the exact value of $\cos\frac{\pi}{12}$.

$$= \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

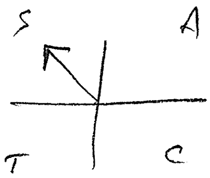
$$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}$$

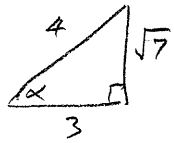
$$= \frac{1+\sqrt{3}}{2\sqrt{2}}$$

OR $\frac{\sqrt{2}+\sqrt{6}}{4}$

2. (5 marks)



Given $\cos\alpha = -\frac{3}{4}$, α obtuse



Determine

(a) $\tan\alpha$ $\quad -\frac{\sqrt{7}}{3}$ [2]

(b) $\sin(\alpha - 45°)$ $= \sin\alpha\cos 45° - \cos\alpha\sin 45°$ [3]

$= \frac{\sqrt{7}}{4}\cdot\frac{1}{\sqrt{2}} + \frac{3}{4}\cdot\frac{1}{\sqrt{2}}$

$= \frac{\sqrt{7}+3}{4\sqrt{2}}$

3. (5 marks)

Determine the exact value of each of the following.

(a) $\cos 75° - \cos 15°$ [2]

$= -2\sin 45°\sin 30°$

$= -2\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{2}$

$= -\frac{1}{\sqrt{2}}$

(b) $\sin\frac{11\pi}{12}\sin\frac{\pi}{12}$ [3]

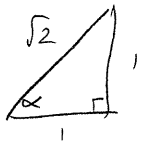
$= \frac{1}{2}\left(\cos\frac{5\pi}{6} - \cos\pi\right)$

$= \frac{1}{2}\left(-\frac{\sqrt{3}}{2} + 1\right)$

$= -\frac{\sqrt{3}}{4} + \frac{1}{2}$

4. (9 marks)

(a) Express $\cos x + \sin x$ in the form $R\cos(x \pm \alpha)$, x radians. [2]

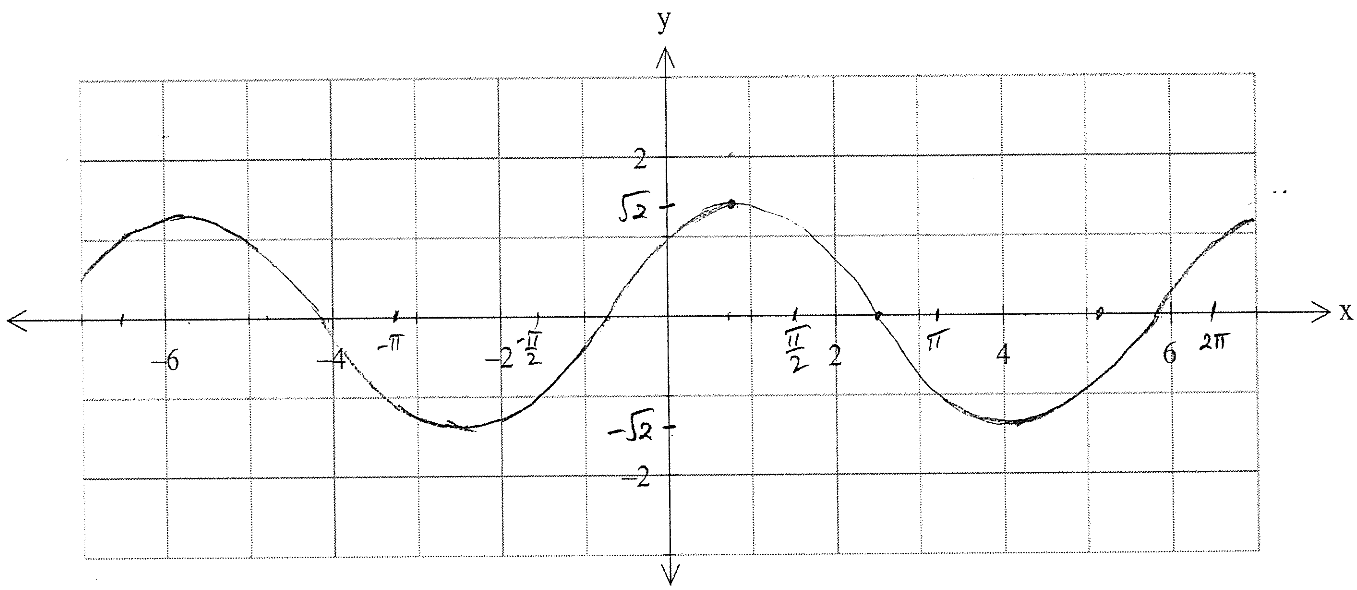


$= \sqrt{2}\left(\frac{\cos x}{\sqrt{2}} + \frac{\sin x}{\sqrt{2}}\right)$

$= \sqrt{2}\cos(x - \alpha)$

$= \sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$

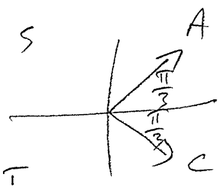
(b) Sketch $y = \cos x + \sin x$ on the axes below. [3]



(c) Solve $\cos x + \sin x = \dfrac{1}{\sqrt{2}}$ [4]

$\sqrt{2}\cos\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\cos\left(x - \frac{\pi}{4}\right) = \frac{1}{2}$



REF ANGLE $= \frac{\pi}{3}$

$x - \frac{\pi}{4} = \frac{\pi}{3}$

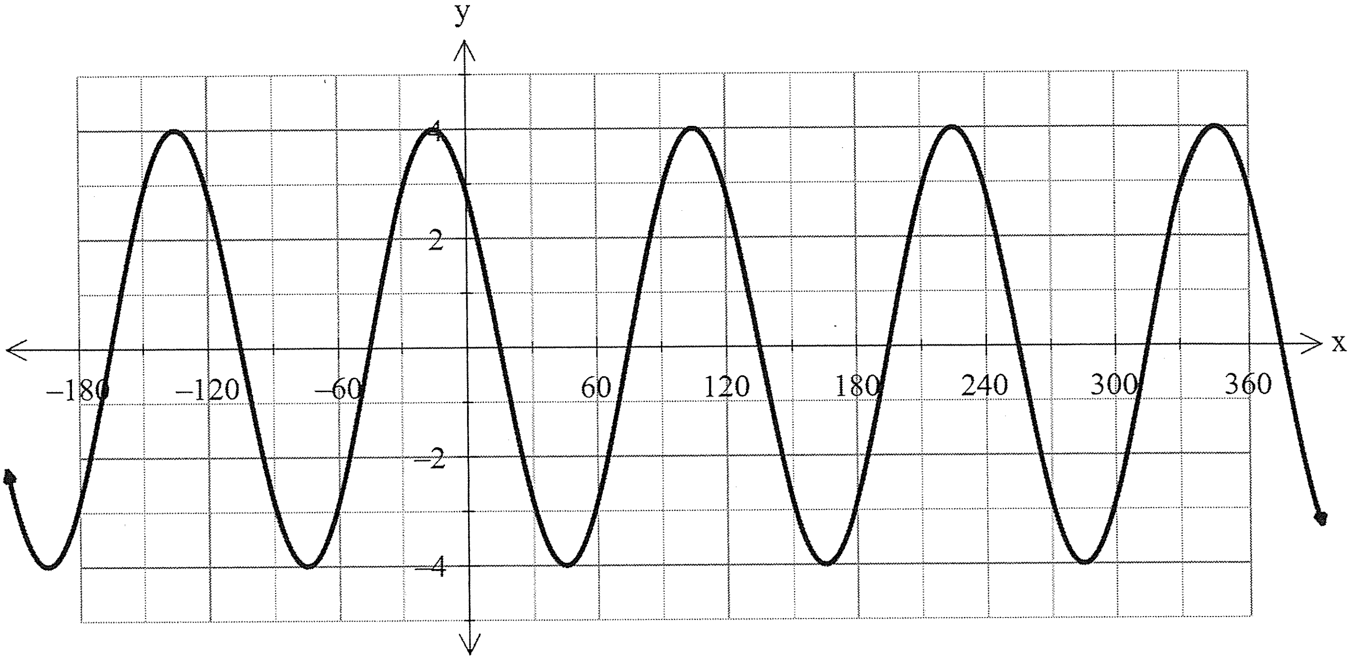
$x = \frac{7\pi}{12} + 2n\pi$

$x - \frac{\pi}{4} = -\frac{\pi}{3}$

$x = -\frac{\pi}{12} + 2n\pi$ $\quad n \in \mathbb{Z}$

5. (4 marks)

Determine the equation of the function shown below, x degrees.

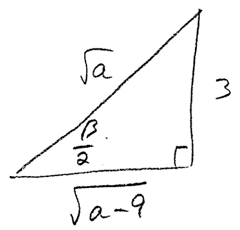


$y = -4\sin(3(x + 15^\circ))$

$y = 4\cos(3(x + 15^\circ))$

6. (3 marks)

Given $\sin\frac{\beta}{2} = \frac{3}{\sqrt{a}}$, show $\sin\beta = \frac{6\sqrt{a-9}}{a}$



$\sin\beta = 2\sin\frac{\beta}{2}\cos\frac{\beta}{2}$

$= 2 \times \frac{3}{\sqrt{a}} \times \frac{\sqrt{a-9}}{\sqrt{a}}$

$= \frac{6\sqrt{a-9}}{a}$

7. (5 marks)

$-\frac{3\pi}{2} \leq 3\vartheta \leq 2\pi$

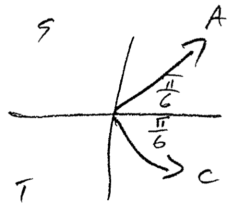
Determine the exact solutions for the equation $\cos(3\vartheta - \frac{\pi}{8}) = \frac{\sqrt{3}}{2}$ for $-\frac{\pi}{2} \leq \vartheta \leq \frac{2\pi}{3}$

$3\vartheta - \frac{\pi}{8} = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{-\pi}{6}, \cancel{\frac{13\pi}{6}}$

$3\vartheta - \frac{3\pi}{24} = \frac{4\pi}{24}, \frac{44\pi}{24}, -\frac{4\pi}{24}, \cancel{\frac{52\pi}{24}}$

$3\vartheta = \frac{7\pi}{24}, \frac{47\pi}{24}, -\frac{\pi}{24}, \cancel{\frac{55\pi}{24}}$

$\theta = \frac{7\pi}{72}, \frac{47\pi}{72}, \frac{-\pi}{72}$



8. (5 marks)

(a) Prove $3\tan 2x + 2\tan x = \frac{8\tan x - 2\tan^3 x}{1 - \tan^2 x}$ for $x \neq \frac{\pi}{4}, \frac{3\pi}{4}$ over $0 \leq x \leq \pi$ [3]

$$LHS = \frac{6\tan x}{1 - \tan^2 x} + 2\tan x$$

$$= \frac{6\tan x + 2\tan x - 2\tan^3 x}{1 - \tan^2 x}$$

$$= \frac{8\tan x - 2\tan^3 x}{1 - \tan^2 x}$$

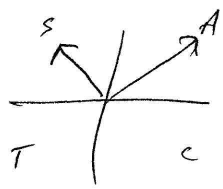
(b) Explain why $x \neq \frac{\pi}{4}, \frac{3\pi}{4}$ in (a). [2]

$1 - \tan^2 x \neq 0$

$1 \neq \tan^2 x$

$\pm 1 \neq \tan x$

$x \neq \frac{\pi}{4}, \frac{3\pi}{4}$



9. (10 marks)

Prove each of the following.

(a) $\frac{1}{2}\sin 2A\cos 2A = \sin A\cos^3 A - \sin^3 A\cos A$ [3]

$RHS = \sin A\cos A\left(\cos^2 A - \sin^2 A\right)$

$= \frac{1}{2}\sin 2A\cos 2A$

(b) $\cot B(\cos B - \sec B) = -\sin B$ [3]

$LHS = \frac{\cos B}{\sin B}\left(\cos B - \frac{1}{\cos B}\right)$

$= \frac{\cancel{\cos B}}{\sin B}\left(\frac{\cos^2 B - 1}{\cancel{\cos B}}\right)$

$= \frac{-\sin^2 B}{\sin B}$

$= -\sin B$

$= RHS$

(c) $\dfrac{\cos 28° + \sin 28°}{\cos 28° - \sin 28°} = \cot 17°$ [4]

$$LHS = \frac{\cos 28^\circ + \cos 62^\circ}{\cos 28^\circ - \cos 62^\circ}$$

$$= \frac{\cancel{\tfrac{1}{2}}\ \cancel{\cos 45^\circ} \cos 17^\circ}{\cancel{\tfrac{1}{2}}\ \cancel{\cos 45^\circ} \sin 17^\circ}$$

$$= \cot 17^\circ$$

$$= RHS$$

10. (5 marks)

Solve $\tan 5x = \cot 13x \quad 0 \le x \le \dfrac{\pi}{4}$

$$\frac{\sin 5x}{\cos 5x} = \frac{\cos 13x}{\sin 13x}$$

$$2 \sin 5x \sin 13x = 2 \cos 5x \cos 13x$$

$$\cancel{\cos 8x} - \cos 18x = \cos 18x + \cancel{\cos 8x}$$

$$0 = \cancel{2} \cos 18x$$

$$18x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}$$

$$x = \frac{\pi}{36}, \frac{3\pi}{36}, \frac{5\pi}{36}, \frac{7\pi}{36}, \frac{9\pi}{36}$$